

Jeans Instability, Jeans Entropy, and the Entropy Origin of Gravity

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Abstract

An entropic origin of gravity is re-visited. Isothermal self-gravitating clouds seen as an ideal gas are analyzed. Gravitational attraction within the isothermal cloud in equilibrium is balanced by the pressure, which is of a pure entropic nature. The notion of the Jeans Entropy of the cloud, corresponding to the entropy of the self-gravitating cloud in mechanical and thermal equilibrium, is introduced. Balance of the gravitational compression and the entropic repulsion yields the scaling relation hinting to the entropic origin of the gravitational force. The analysis of the Jeans instability enables elimination of the “holographic screen” or “holographic principle” necessary for grounding of the entropic origin of gravity.

Keywords: Gravity; Entropic force, Jeans instability, Isothermal self-gravitating cloud.

1. INTRODUCTION

The origin and physical nature of gravity was subjected to the stormy scientific discussion recently. The discussion was started by the paper by Eric Verlinde in which Newton’s law of gravitation appeared within a theory in which space emerges through a holographic scenario [1]. Gravity is identified with an entropic force caused by changes in the information associated with the positions of material bodies [1, 2]. Entropic force acting in a physical system is an emergent phenomenon resulting from the entire system’s statistical tendency to increase its entropy, rather than from a particular underlying force on the atomic scale [3, 4].

Brownian motion provides a basic example of entropic forces emerging from microscopic fluctuations [3]. Entropic force emerges in the linear system of elementary non-interacting magnets/spins exposed to the magnetic field [4].

The entropic origin of gravity was clarified by Verlinde in the recent paper in which it was demonstrated that the observed dark energy and the phenomena traditionally attributed to dark matter have a common origin and are connected to the emergent nature of space-time and gravity [5]. The entropic origin of gravity was discussed and developed in refs. 6, 7. In particular, it was shown, that classical Newtonian gravity may be interpreted in terms of an entropic force; it was also shown that the fact that Newtonian gravity is described by a conservative force places significant constraints on the form of the entropy and temperature functions [6].

The entropy origin of gravity was criticized in refs. 8–10. Kobakhidze argued that experiments with ultra-cold neutrons in the gravitational field of Earth disprove the entropic origin of gravitation [8]. Gao argued, in turn, that neither holographic screen nor test particle satisfies all requirements for the existence of entropic force in a thermodynamics system [9]. Yang suggested that gravity may be seen as entropic force only for systems with constant temperature and zero chemical potential [10].

We conjecture that that the weakest point in the argumentation by Verlinde, resulting in the relating of gravity to entropy forces is the “holographic screen” which seems to be a fanciful intellectual construction, to be eliminated due to the “Occam razor”. We demonstrate that the analysis of the Jeans gravitational instability [11–14] supports the entropic origin of the gravity and enables elimination of the “holographic principle”.

2. RESULTS

2.1 Self-gravitating system with the constant mass

The instability of self-gravitating systems was first studied by Jeans in non-relativistic Newtonian gravity [11]. It was extended to General Relativity by Evgeny Lifshitz in 1946 and it is broadly used in modern cosmology to study the evolution of density perturbations in the expanding universe [12, 15, 16]. We demonstrate that re-interpretation of the Jeans instability enables new insights in the entropy nature of gravity.

Consider the self-gravitating spherical cloud with the mass of M and radius r . The total self-gravitational energy of the cloud, is calculated in a way resembling the calculation of the Born energy of the electrically charged particle (ion) [17]. Imagine the process of gravitational cloud formation by gradually increasing its mass from zero to its full mass M . At any stage of this process let the mass of the cloud be m and let this be incremented by dm . The work done in bringing this additional mass, and therefore the total change in the potential energy of the cloud is given by Eq. 1:

$$\Delta U_{gr} = \int_0^M G \frac{m dm}{r} = \frac{GM^2}{2r}, \quad (1)$$

where G is the gravitational constant. Again, the resemblance of Eq. 1 to the expression obtained for the Born energy of the charged particle should be emphasized [17]. Jeans instability emerges in the self-gravitating cloud when perturbations occurring on the sufficiently large spatial scale become unstable; such “large” instabilities cause the matter in the cloud to clump together and lead to gravitational collapse [11–16]. The traditional approach to the analysis of the Jeans instability implies treatment of the self-gravitating cloud as a continuous liquid. Combining of the Euler equation, continuity equation and the state equation of the self-gravitating liquids yields the following result, the perturbations with $\lambda > \lambda_J$ become unstable, where the Jeans length λ_J is supplied by Eq. 2:

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho_0}}, \quad (2)$$

where c_s is the adiabatic sound speed, and ρ_0 is the density of the unperturbed self-gravitating liquid. The Jeans mass M_J is the mass contained in a Jeans sphere of radius $R_J = \frac{1}{2}\lambda_J$. Note, that

the traditional approach to the Jeans instability implies isentropic (adiabatic) flows, i.e. $S = const$ where S is the entropy of the self-gravitating liquid [9–14].

In our treatment of the Jeans instability we adopt the simple qualitative approach. Consider the isothermal ($T = const$) spherical homogeneous self-gravitating cloud built of the identical particles; this assumption is not correct for the collapse of self-gravitating clouds, forming stars, which is usually supposed to be adiabatic [11–15]. Thus, we consider the pure *Gedanken Experiment* in which the self-gravitating cloud is seen as an isothermal ideal gas. Two opposing physical factors control the behavior of the cloud. The gas pressure, caused by the thermal movement of the particles comprising the cloud, tries to expand the cloud, whereas gravitation forces the cloud to collapse. The balance of these factors controls the physical behavior of the cloud. Compression the cloud to a radius $r - dr$ demands work dA done against the gas pressure, given by Eq. 3:

$$dA = -p_{en}dV = p_{en}4\pi r^2 dr, \quad (3)$$

where p_{en} is the “entropy pressure”. Labeling of the pressure as the “entropy one” emphasizes the assumption implying that our self-gravitating cloud behaves as an isothermal ideal gas. Recall, that the pressure emerging under isothermal compression of an ideal gas is of a pure entropic nature: indeed, the internal energy of the ideal gas under isothermal conditions remains unchanged, and work done against the gas pressure is expressed as $dA = \Delta F = -p_{en}dV = -TdS$, where ΔF is the Helmholtz free energy of the cloud [18]. The entropic pressure is calculated according to Eq. 4:

$$p_{en} = \frac{F_{en}}{4\pi r^2} = \frac{1}{4\pi r^2} T \frac{dS}{dr}, \quad (4)$$

where $F_{en} = T \frac{dS}{dr}$ is the entropic force [3, 4, 18, 19], T and S are the temperature and entropy of the cloud respectively. During the compression, the gravitational energy is released. When this energy equals the amount of work to be done on the gas, the critical (Jeans) mass is attained. Equating the work to the change in the gravitational energy yields:

$$dA = dU_{gr} \quad (5)$$

$$T \frac{dS}{dr} = \frac{GM^2}{2r^2} \quad (6)$$

The mass of the cloud is supposed to be constant, $M = const$; thus, integration of Eq. 6 immediately yields

$$S(r) = -\frac{GM^2}{2Tr} + S_0 \quad (7)$$

where S_0 is the constant of integration, to be established from the Sackur-Tetrode equation (see ref. 20). Eq. 7 is easy to understand intuitively, indeed: the gravitation-inspired contribution to entropy, namely $S(r) - S_0 = -\frac{GM^2}{2Tr}$ is negative and it tends to diminish the entropy of the cloud, under its compression. It is seen from Eq. 7 that when the mass of the cloud M is constant $\lim_{r \rightarrow \infty} S(r) = S_0$ takes place. This means that S_0 may be calculated from the low concentration, rarefied limit of thermal state of the self-gravitating cloud. The Sackur-Tetrode entropy of the ideal gas is supplied by Eq. 8:

$$S_J = k_B N \ln \left(\frac{V}{N \lambda_{th}^3} \right) + \frac{5}{2}, \quad (8)$$

where k_B is the Boltzmann constant, N is the total number of particles in the cloud, V the volume occupied by the cloud, and λ_{th} is the thermal wavelength, which for the ideal gas is supplied by Eq. 9 (see ref. 18):

$$\lambda_{th} = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}, \quad (9)$$

where m is the mass of the particle, and \hbar is the Planck constant. In the rarefied, low concentration, classical limit $\frac{V}{N\lambda_{th}^3} \gg 1$ takes place; thus, Eq. 10, constituting constant S_0 is derived

$$\lim_{r \rightarrow \infty} S_J = S_0 \cong k_B N \ln \left(\frac{V}{N\lambda_{th}^3} \right) = -k_B N \ln \left(n\lambda_{th}^3 \right) \cong 3k_B N \ln \frac{r}{N\lambda_{th}} > 0, \quad (10)$$

where $n = \frac{N}{V}$ is the concentration of particles in the cloud and $\frac{r}{N\lambda_{th}} \gg 1$ is assumed. Let us go back to Eq. 7: denoting the Jeans entropy change $\Delta S_J = S(r) - S_0$, we obtain:

$$\Delta S_J = -\frac{GM^2}{2Tr} = -\frac{\Delta U_{gr}}{T} \quad (11)$$

$$|\Delta S_J| = \frac{\Delta U_{gr}}{T} \quad (12)$$

Equations 11 and 12 hint to the entropic origin of gravity. Consider, that they were derived without involving the holographic screen. Obviously, the Jeans entropy change $\Delta S_J = S(r) - S_0$ given by Eq. 11 is diminished with the decrease of the radius of the self-gravitating cloud of the constant mass.

2.2 Self-gravitating system with a variable mass and constant density

Consider now the open self-gravitating isothermal system with a variable mass $M(r)$ and radius r (a cloud, which may absorb or emanate particles). Assume, that the spherical shape of the cloud and its temperature T and density ρ_0 remain constant. Arriving or emanated particles will change the entropy of the cloud; thus, giving rise to the “entropic force”. Considering in this case $M(r) = \rho_0 \frac{4}{3}\pi r^3$ and substitution into Eq. 6 gives rise to Eq. 13 taking place in equilibrium:

$$T \frac{dS}{dr} = \frac{8\pi^2}{9} G \rho_0^2 r^4 \quad (13)$$

Integration of Eq. 13 results in Eq. 14:

$$S(r) = \frac{8\pi^2}{45} G \rho_0^2 \frac{r^5}{T} + S_{01}, \quad (14)$$

where S_{01} is the constant of integration, to be established now from the initial thermodynamic state of the cloud (for the rarefied clouds it is calculated from the Sackur-Tetrode equation, as demonstrated above). Denoting the Jeans entropy change $\Delta S_J = S(r) - S_{01}$, considering $M(r) = \rho_0 \frac{4}{3}\pi r^3$, and involving Eq. 1 results in the scaling relations supplied by Eqs. 15-16:

$$\Delta S_J \sim \frac{GM^2(r)}{Tr} \quad (15)$$

$$\Delta S_J \sim \frac{\Delta U_{gr}}{T} \quad (16)$$

Taking along carefully the constants emerging from integration yields the exact Equations 17-18:

$$\Delta S_J = \frac{GM^2(r)}{10Tr} \quad (17)$$

$$\Delta S_J = \frac{\Delta U_{gr}}{5T} \quad (18)$$

Scaling laws supplied by Eqs. 15-18 resemble those given by Eq. 7 and Eq. 12, again hinting to the entropic origin of gravity. The equilibrium (Jeans) change in the entropy of the isothermal, self-gravitating, open cloud, keeping its density constant, increases with the growth of its radius (see Eq. 14). This conclusion is easily understood intuitively; indeed, increase in the radius of the cloud, keeping the density constant, results in the increase in the gravitational compression, which scales now as $\sim r^4$ (see Eq. 13), which, should be compensated in equilibrium by the entropy-inspired expansion of the cloud.

2.3 Interaction between gravitational spherical clouds

Consider two equilibrium, isothermal spherical clouds of the constant mass M and radius r contacting one another (their centers are separated by the distance $2r$) and interacting on the “short” time scale, defined by: $t \ll \tau = \sqrt{\frac{r^3}{GM}}$, providing the initial spherical shape and density of the clouds untouched. The total Jeans entropy of two clouds is $\Delta S_{J,total} = -\frac{GM^2}{Tr}$ (see Eq. 11). The gravitational force acting between clouds equals: $f_{gr} = \frac{GM^2}{4r^2}$; thus, the gravitational force is easily re-written as:

$$f_{gr} = \frac{1}{4}T \frac{\partial \Delta S_{J,total}}{\partial r}. \quad (19)$$

Equation 19 within an accuracy of a numerical coefficient is an equation describing the temperature-dependent entropic forces [3, 4, 18, 19].

3. DISCUSSION

Analysis of the Jeans instability in the isothermal self-gravitating clouds, seen as an ideal gas shows that the equilibrium Jeans entropy of the cloud scales as $|\Delta S_j| \sim \frac{\Delta U_{gr}}{T}$ where ΔU_{gr} is the total self-gravitational energy of the cloud; for the clouds with a constant mass the exact relation $|\Delta S_j| = \frac{\Delta U_{gr}}{T}$ is true. This expression hints to the entropy origin of the gravity. The analysis of the Jeans instability enables to eliminate the “holographic principle” involved for the grounding of the entropy origin of the gravitational force. The gravity force acting between two isothermal equilibrium clouds scales in turn as: $f_{gr} \sim T \frac{\partial \Delta S_j}{\partial r}$, which is typical for entropic forces [3, 4, 18, 19]. Hence, the analysis of the isothermal Jeans instability supports the “entropic origin” of gravity. Entropy of the physical system is its extensive characteristics [20]; thus, the entropy force may be expressed as: $f_{en} = TM \frac{\partial S^*}{\partial r}$, where M is the mass of the system, and S^* is the specific entropy (entropy per unit mass). Thus, we recognize that the triad of fundamental forces, namely: the gravitational force, the inertia force

and the entropy force are proportional to the mass of the physical system. This fact enables re-consideration and re-shaping of the Einstein equivalence principle.

4. CONCLUSIONS

The novel approach to gravity, seen as an emerging force appearing as a consequence of the information associated with the positions of material bodies revolutionized the foundations of physics [1, 5–7]. This idea, suggested by Eric Verlinde, exploits explicitly the “holographic principle”, implying that the description of a volume of space can be thought of as encoded on a lower-dimensional boundary to the region (labeled as the “holographic screen”) [21]. The analysis performed by Verlinde and developed by other investigators [1, 2, 5–7] demonstrated that gravity is actually a kind of “entropic phenomena”, i.e. forces resulting from the entire system’s statistical tendency to increase its entropy, rather than from a particular underlying fields/interactions acting between physical objects. This idea was criticized in refs. 8–10 and it is now subjected to intensive and fruitful scientific discussion. We suggest an alternative approach to the problem, based on the analysis of the Jeans gravitational instability. Jeans instability in self-gravitating clouds appears as an interplay of two opposite tendencies, namely: gravity induced compression and thermal expansion due to the random motion of the particles constituting the cloud. We considered isothermal self-gravitating clouds. In the isothermal clouds (whatever is their physical nature) the thermal pressure emerges as a pure entropic phenomenon; indeed, the internal energy of the isothermal cloud remains constant [20]. Thus, mechanical equilibrium of the gravitational cloud is seen as a balance of the gravity-induced compression and entropy-driven expansion. We introduced the notion of the “Jeans Entropy” denoted ΔS_J , i.e. the entropy of the isothermal self-gravitating cloud in thermal and mechanical equilibrium. The analysis of the equilibrium shows that the Jeans entropy of the cloud scales as $|\Delta S_J| \sim \frac{\Delta U_{gr}}{T}$ where ΔU_{gr} is the total self-gravitational energy of the cloud; moreover, for the clouds with the constant mass $|\Delta S_J| = \frac{\Delta U_{gr}}{T}$ takes place exactly. This expression supports the “entropic origin” of gravity; however, our analysis enables elimination of the “holographic principle” [21]. We also consider gravity interaction between two isothermal equilibrium isothermal clouds; the Newton gravity force scales in this case as $f_{gr} \sim T \frac{\partial S_J}{\partial r}$, which is typical for entropic forces. We conclude that the analysis of the Jeans instability sheds light on the entropic nature of the gravity force, at least in the isothermal situation, addressed in ref. 9. It should be emphasized, that the triad of fundamental forces, namely: the gravitational force, the inertia force and the entropy force are proportional to the mass of the physical system. This fact enables re-consideration of the Einstein equivalence principle.

5. AUTHOR CONTRIBUTIONS

Conceptualization, E.B.; methodology, E.B.; investigation, E.B. writing—original draft preparation, E. B.; writing—review and editing, E. B.

All authors have read and agreed to the published version of the manuscript.

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7. DATA AVAILABILITY STATEMENT

The data used to support the findings of this study are available from the corresponding author upon request.

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9. CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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