

Theoretical Calculation of the Gravitational Constant Using The Elementary Charge, Speed of Light, Z Boson Mass, and Relativistic Mechanics

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Abstract

The gravitational constant is an unchanging quantity included in the law of universal gravitation. The value of this constant is known much less accurately than other fundamental physical constants, which is due to the experimental difficulties of measuring it. Here we report a simple method for the theoretical calculation of the gravitational constant using the elementary charge, speed of light, Z boson mass, and relativistic mechanics.

Keywords: Elementary charge, Gravitational constant, Stoney mass, Z boson

There are no infinite quantities in physics at all.
In math, yes, infinities exist, but not in physical reality.

1. INTRODUCTION

The gravitational constant (G) is the proportionality constant in Newton's law of universal gravitation. The value of this constant is known to be much less accurate than that of other fundamental physical constants, and the results of experiments to refine it continue to vary. This is because the gravitational force is extremely weak compared with other fundamental forces. Therefore, the problems are caused by experimental difficulties in measuring small forces taking into account a large number of external factors. Despite the increase in the measurement accuracy of G , the results of experiments on its refinement continue to differ. Therefore, we can only say that the values of the gravitational constant measured by various methods over the past decade [1–5], lie in the interval $(6.67 \div 6.676) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. The discrepancy in G calculations is such a serious problem that it is considered as a metrological and scientific dead end. But it turns out that there is an unexpected solution to this problem: we can theoretically calculate the gravitational constant.

To solve this problem, in particular, it is necessary to use the value of an elementary electric charge. However, the charge has the different numerical value and dimension in the different systems of measurement.

The most widely used in physics is the International System of Units (SI) composed of other sub-systems, in particular, the m-kg-s system of mechanical units (called the MKS system) and the m-kg-s-A system of electromagnetic units (called the MKSA system). Second system differs first system primarily in that, along with the existing three basic units (meter, kilogram, second), it has a fourth basic unit – ampere (A).

In the MKSA system, the elementary charge $e = 1.6 \times 10^{-19}$ C, and the proportionality coefficient, included in Coulomb's law, $k = 9 \times 10^9$ N m² C⁻².

In 2018, we showed in paper [6], that the electromagnetic units of the MKSA system (A, C, V, Ω , etc.) can be written using the basic units of the MKS system: m, kg, s. In particular, it is shown that in the MKS system

$$e = 1.6 \times 10^{-25} \text{ kg m s}^{-1}, \quad (1)$$

$$k = 9 \times 10^{21} \text{ m kg}^{-1}. \quad (2)$$

Here it is necessary to recall one historical fact.

In 1874 in Belfast, the Irish physicist J. Stoney presented his famous report [7], in which he proposed the first natural system of units of mass (m_s), length (l_s), and time (t_s), built on the fundamental constants c ($c = 3 \times 10^8$ m s⁻¹ is the speed of light in vacuum), G , and e . The modern meanings of the Stoney units are as follows:

$$m_s = (ke^2/G)^{1/2} = 1.859 \times 10^{-9} \text{ kg}, \quad (3)$$

$$l_0 = (ke^2G/c^4)^{1/2} = 1.38 \times 10^{-36} \text{ m}, \quad (4)$$

$$t_0 = (ke^2G/c^6)^{1/2} = 4.6 \times 10^{-45} \text{ s}, \quad (5)$$

where recommended value $G = 6.6743 \times 10^{-11}$ m³ kg⁻¹ s⁻² [8].

However, physicists failed to understand the significance of Stoney system for science. It is now clear that this was a mistake, because using this system immediately gives important results.

In 2021, we showed in article [9], that the Stoney mass m_s separates the macrocosm and microcosm; i.e, this mass is the minimum value for the masses of ordinary bodies and the maximum value for the masses of elementary particles. This means the following.

On the one hand, an ordinary body with such a minimum mass would have the smallest *gravitational radius* $R_g = m_s G/c^2 = 1.38 \times 10^{-36}$ m.

(In educational literature, the gravitational radius of a body $R_g = mG/c^2$ is mistakenly confused with its *Schwarzschild radius* $R = 2R_g = 2mG/c^2$; in fact, these are completely different physical quantities [9]).

On the other hand, a charged elementary particle with such a maximum relativistic mass would have the smallest *classical radius* $R_0 = ke^2/m_s c^2 = 1.38 \times 10^{-36}$ m.

Thus, the modern values of the Stoney length and the Stoney time, l_0 and $t_0 = l_0/c$, are the *elementary length* and the *elementary time*, i.e. the smallest values of length and time that exist in nature. Moreover, it turned out that these quantities are associated with the so-called extra dimensions.

The space around us has three dimensions: length, width and height. Scientists consider these dimensions as independent coordinates that are necessary to describe the position of any point in a three-dimensional (3D) space and denote them by x^1, x^2, x^3 .

The theorists have suggested that in addition to the usual three dimensions, there are additional 6 dimensions that are curled up in a circle of microscopic radius and, therefore, cannot be detected directly.

In 2022, in the paper [10], the idea was put forward that in three-dimensional space a *physical point* is, in fact, a minuscule *ball*. It is shown, that any additional spatial dimension x^D ($D > 3$) is the physical point radius r , which can be expressed in terms of the elementary length l_0 or the product of the speed of light in a vacuum c and the elementary time t_0 :

$$x^D(D > 3) = r = l_0 \text{ or } x^D(D > 3) = r = ct_0. \quad (6)$$

Even more unexpected is that relativistic mechanics is a necessary part of our research.

The relativistic mechanics, as a special branch of physics, arose in 1901, when the German researcher W. Kaufmann discovered an amazing fact in experiments on the study of cathode particles (electrons): the ratio of the electron charge to its mass e/m decreased with increasing speed [11]. Thus, Kaufman's experiments clearly indicated an increase in the mass of a moving electron. It was a discovery that classical physics could not explain. The mass of the electron turned out to depend on the speed, which in these experiments reached 95% of the speed of light in vacuum. Further research showed that this dependence is well described by the formula

$$m = m_0/(1 - v^2/c^2)^{1/2}, \quad (7)$$

where m_0 is the mass of a rest particle, m is its relativistic mass, v is speed.

Accordingly, the mass (energy) of other particles also depends on the speed. We see this in cosmic rays, which contain particles of different energies.

In 1966, American scientist K. Greisen [12] and, independently, Soviet physicists G. Zatsepin and V. Kuzmin [13], proposed theoretical upper limit to the energy of charged particles (protons) coming to the Earth from space (GZK limit): 5×10^{19} eV. Scientists have suggested that the limit is set by slowing-interactions of the protons with the microwave background radiation over long distances. However, this suggestion contradicts to the astrophysical observations: in cosmic rays was detected the extreme-energy protons with energy $(1 \div 3) \times 10^{20}$ eV [14, 15]. The observed existence of these particles is called GZK paradox. Thus, these facts indicate that the energy of a moving charged particle (particularly, of proton) is rising only to specific value. The solution of this problem will allow us to better understand the mysterious world of elementary particles.

2. NEW FORMULAS AND THE SCOPE OF APPLICABILITY OF RELATIVISTIC MECHANICS

As is known since the beginning of the 20th century, the energy of a moving particle, $E = mc^2$, is increased with velocity v :

$$mc^2 = m_0c^2/(1 - v^2/c^2)^{1/2}, \quad (8)$$

where m_0c^2 is the rest particle energy.

An assumption arose, which then turned into the assertion that when the particle's velocity approaches the speed of light ($v \rightarrow c$) the particle's energy increases without bound ($mc^2 \rightarrow \infty$). However, this is simply a historical delusion that is easy to disprove.

Firstly, this postulate contradicts astrophysical observations: in the universe we do not see moving particles with infinite energy.

Secondly, if the speed of the particle is equal to the speed of light, $v = c$, then the denominator in equation (8) becomes zero, and this equation itself becomes invalid, because we know from arithmetic that it is impossible to divide by zero.

Thirdly, we can rewrite the equation (8) in the form:

$$mc^2 = (m^2v^2c^2 + m_0^2c^4)^{1/2}. \quad (9)$$

Immediately evident that if the velocity $v = c$, this equation becomes invalid because

$$mc^2 \neq (m^2c^4 + m_0^2c^4)^{1/2}. \quad (10)$$

Consequently, equations (8) and (9) are valid if the velocity v does not exceed a certain value v_M , which is very close to the speed of light: $v \leq v_M$, $v_M \approx c$.

Thus, it is clear that the energy of a moving particle cannot increase infinitely, and it has reached the maximum value $E_M = \lim mc^2 = Mc^2$ on the certain velocity v_M :

$$Mc^2 = m_0c^2 / (1 - v_M^2/c^2)^{1/2}. \quad (11)$$

According to the relativistic mechanics, along the direction of motion of a charged elementary particle decrease its linear dimensions, including classical radius R :

$$R = ke^2 / m_0c^2. \quad (12)$$

As we mentioned, in the MKS system $e = 1.6 \times 10^{-25}$ kg m s⁻¹, $k = 9 \times 10^{21}$ m kg⁻¹, therefore this equation can be written as:

$$R = km_0(e/m_0c)^2 = km_0n, \quad (13)$$

where a dimensionless factor

$$n = (e/m_0c)^2. \quad (14)$$

Thus, the factor n is inversely proportional to the rest mass of a charged elementary particle.

For instance, for a proton the factor

$$n_p = (e/m_p c)^2 = 1.02 \times 10^{-13} \quad (15)$$

(the rest proton mass $m_p = 1.67 \times 10^{-27}$ kg). Further, from equation (12) turns out:

$$ke^2 = Rm_0c^2 = Rn(m_0c^2/n) = rMc^2, \quad (16)$$

where $r = Rn$,

$$Mc^2 = m_0c^2/n \quad (17)$$

or

$$M = m_0/n = m_0^3c^2/e^2. \quad (18)$$

Therefore, considering equation (11), factor

$$n = m_0c^2/Mc^2 = (1 - v_M^2/c^2)^{1/2}. \quad (19)$$

Hence, it is easy to define the limits of velocity ($v_M = \lim v$), momentum ($p_M = \lim p = \lim m v$) and energy ($E_M = \lim E = \lim m c^2$) of a moving charged elementary particle [16]:

$$v_M = c(1 - n^2)^{1/2} \text{ (i.e., } v_M \approx c), \quad (20)$$

$$p_M = M v_M = m_0 v_M / n \approx m_0 c / n, \quad (21)$$

$$E_M = M c^2 = m_0 c^2 / n. \quad (22)$$

For example, the maximum speed of a proton

$$v_M = c(1 - n_p^2)^{1/2} = c[1 - (1.02 \times 10^{-13})^2]^{1/2} = c(1 - 1.0404 \times 10^{-26})^{1/2}, \quad (23)$$

and its the maximum energy

$$M_p c^2 = m_p c^2 / n_p = 9.19 \times 10^{21} \text{ eV} \quad (24)$$

($m_p c^2 = 0.938 \times 10^9$ eV is the rest proton energy).

This is the upper limit of the cosmic ray spectrum; it is consistent with observations and at is two orders of magnitude exceeds the GZK limit.

Now we can write down the general formulas:

$$m = m_0 / (1 - v^2/c^2)^{1/2}, v \leq v_M, m \leq M, M \leq m_s; \quad (25)$$

$$E = m c^2 = m_0 c^2 / (1 - v^2/c^2)^{1/2}, v \leq v_M, m \leq M, M \leq m_s; \quad (26)$$

$$p = m v = m_0 v / (1 - v^2/c^2)^{1/2}, v \leq v_M, m \leq M, M \leq m_s; \quad (27)$$

$$p = m_0 v, \text{ if } m_0 \geq m_s. \quad (28)$$

These formulas reflect the obtained results, namely:

- equations (25), (26) and (27) show the existence of limits for the speed, momentum and energy of a stable charged elementary particle, and the limit of the relativistic mass cannot exceed the Stoney mass;
- equation (28) reflects the invariability (constancy) of the mass for all bodies whose mass exceeds the Stoney mass.

Hence a very important conclusion: the area of applicability of relativistic mechanics is limited to the microcosm and does not extend to the macrocosm, i.e. the ordinary bodies (whose rest mass exceeds the Stoney mass) obey the laws of the classical mechanics.

We can now set out a method for the theoretical calculation of the gravitational constant.

3. METHOD

According to the results obtained in paper [9] and equations (25)–(27), the absolute limit of the relativistic mass of a charged elementary particle is equal to the Stoney mass: $\lim M = m_s$. Therefore, it is possible to calculate the limiting rest mass, $\lim m_0 = m_m$, that a stable charged elementary particle can have (it is convenient to call it “*maximon*”; this name for an elementary particle with a maximum rest mass was proposed by the Soviet physicist M. Markov in 1965 [17]).

Substituting in equation (18) $M = m_s$ and $m_0 = m_m$, we get:

$$m_s = m_m^3 c^2 / e^2 \quad (29)$$

($e = 1.6 \times 10^{-25}$ kg m s⁻¹). Hence,

$$m_m = (m_s e^2 / c^2)^{1/3} = 8.0979 \times 10^{-26} \text{ kg}. \quad (30)$$

This is the theoretical maximum rest mass that stable charged elementary particles (maximons) can have; rest energy of such particles $E_m = m_m c^2 / e = 45.426$ GeV ($ee = 1.6 \times 10^{-19}$ C).

Note that we calculated the mass (energy) of the maximon using the Stoney mass, according to the recommended value $G = 6.6743 \times 10^{-11}$ m³ kg⁻¹ s⁻² [8].

Thus, on the contrary, if we knew the exact value of the mass of the maximon m_m , then we could easily calculate the exact value of the Stoney mass m_s using formula (29), and then calculate the exact value of the gravitational constant:

$$G = k e^2 / m_s^2. \quad (31)$$

Naturally, the question arises: do the maximons exist?

The affirmative answer is due to the fact that the total energy of the maximon and antimaximon is: 2×45.426 GeV = 90.852 GeV. This needs to be explained.

In 1983, at CERN (European Organization for Nuclear Research) neutral particles (called Z bosons) predicted by the electroweak theory were discovered. These particles were born in the collision of colliding beams of protons and antiprotons. In a very short time ($\sim 3 \times 10^{-25}$ s) Z bosons decay in a fermion and its antiparticle (in particular, in an electron and a positron), flying out in opposite directions.

The theory predicted a Z boson energy of about 90 GeV; the experiment gave a close value.

Based on further experiments, current estimate of the Z boson energy was made: $E_Z = 91.1876$ GeV [18]. Hence it follows that the decay of the Z boson, in particular, produces the electrons and positrons with rest energy $E_0 = E_Z / 2 = 45.5938$ GeV; this energy corresponds to the rest mass $m_0 = E_0 e / c^2 = 8.1278 \times 10^{-26}$ kg ($ee = 1.6 \times 10^{-19}$ C).

As we can see, the value of the mass m_0 (energy E_0) of the electron and the positron measured in these experiments is slightly larger than the calculated value of the mass (energy) of the maximon ($m_m = 8.0979 \times 10^{-26}$ kg, $E_m = 45.426$ GeV); this discrepancy leads us to the following conclusions.

4. CONCLUSIONS

1. The currently accepted value of the Z boson energy (91.1876 GeV), is slightly overestimated; this means that a system error was made in estimating the energy of the electron and positron produced during its decay (apparently, the increase in energy during their movement is not fully taken into account).

For charged particle with energy $E_0 = 45.5938$ GeV and rest mass $m_0 = 8.1278 \times 10^{-26}$ kg, the factor $n = (e/m_0c)^2 = 4.3165 \times 10^{-17}$, and the relativistic mass limit $M = m_0/n = 1.883 \times 10^{-9}$ kg.

In this case, the gravitational constant $G = ke^2/M^2 = 6.506 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$; this value is completely inconsistent with the experimental values of this constant, which are lying within $(6.67 \div 6.676) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$.

Thus, it is necessary to radically analyze the method for estimating the Z boson energy.

2. Having determined the exact value of the energy (mass m_m) of the electron and positron produced during the decay of the Z boson, we will can easily calculate the exact value of the Stoney mass m_s using formula (29), and then calculate the exact value of the gravitational constant using formula (31).

5. COMPETING INTERESTS

The author declare no competing interests.

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